[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1223 **F**

Unique Paper Code : 2352011202

Name of the Paper : CALCULUS

Name of the Course : **B.Sc. (H) Mathematics UGCF-2022**

Semester : II – DSC 5

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **three** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) State and prove the sequential criterion for the limit of a real valued function. (5)

   (b) Use $\epsilon - \delta$ definition of limit to establish the following limit : (5)

$$\lim_{x \to 2} \frac{1}{1-x} = -1.$$

(c) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as (5)

$$f(x)=\begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Show that f has a limit only at $x = 0$.

(d) Let $A \subseteq \mathbb{R}$, let $f: A \to \mathbb{R}$, and let $c \in \mathbb{R}$ be a cluster point of A. If $\lim_{x \to c} f > 0$, then show that $f(x) > 0$ for all $x \in A \cap V_\delta(c)$, $x \neq c$. (5)

2. (a) If f is continuous at x_0 and g is continuous at $f(x_0)$ then prove that the composite function $g \circ f$ is continuous at x_0. (5)

(b) Let $f(x) = \frac{1}{x}\sin\frac{1}{x^2}$ for $x \neq 0$ and $f(0) = 0$. Show that f is discontinuous at 0. (5)

(c) State Intermediate Value Theorem. Prove that $xe^x = 1$ for some x in $(0,1)$. (5)

(d) Let f be a continuous real-valued function with domain (a, b). Show that if $f(r) = 0$ for each rational number r in (a, b), then $f(x) = 0$ for all $x \in (a, b)$. (5)

3. (a) Prove that if a real valued function f is continuous on $[a, b]$ then it is uniformly continuous on $[a, b]$. (5)

(b) Show that the function $f(x) = \frac{1}{x}$ is uniformly continuous on (a, ∞) for $a > 0$ but it is not uniformly continuous on $(0,1)$. (5)

(c) Let $f(x) = |x| + |x - 1|$, $x \in \mathbb{R}$. Draw the graph and give the set of points where it is not differentiable. Justify also. (5)

(d) Prove that if f and g are differentiable on $\mathbb{R}$, if $f(0) = g(0)$ and if $f'(x) \leq g'(x)$ for all $x \in \mathbb{R}$, then $f(x) \leq g(x)$ for $x \geq 0$. (5)

4. (a) State and prove Mean Value Theorem. (5)

(b) State Intermediate Value Theorem for derivatives. Suppose f is differentiable on $\mathbb{R}$ and $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that $f'(x) = \frac{1}{2}$ for some $x \in (0,2)$.

(ii) Show that $f'(x) = \frac{1}{7}$ for some $x \in (0,2)$. (5)

(c) Prove that $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$. (5)

(d) Let f be defined on $\mathbb{R}$ and suppose that $|f(x) - f(y)| \le (x-y)^2$ for all $x, y \in \mathbb{R}$. Prove that f is a constant function. (5)

5. (a) Let f be differentiable function on an open interval (a, b). Then show that f is increasing on (a, b) if $f'(x) \ge 0$. (5)

(b) If $y = e^{\tan^{-1}x}$, prove that (5)

$$(1+x^2)y_{n+2} + (2(n+1)x - 1)y_{n+1} + n(n+1)y_n = 0.$$

(c) If $y = \cos(m \sin^{-1} x)$, find $y_n(0)$. (5)

(d) Stating Taylor's theorem find Taylor series expansion of e^x. (5)

6. (a) Find $\lim_{x \to +\infty} \left[x - \ln\left(x^2+1\right)\right]$. (5)

(b) Determine the position and nature of the double points on the curve (5)

$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0.$

(c) Sketch a graph of the rational function showing the horizontal, vertical and oblique asymptote (if any) of $y = \frac{x^2-2}{x}$. (5)

(d) Sketch the curve in polar coordinates of $r = \sin 2\theta$. (5)

(1000)

[This question paper contains 4 printed pages.]

Your Roll No...............

E

Sr. No. of Question Paper : 5776

Unique Paper Code : 42351201

Name of the Paper : Calculus and Geometry, CBCS (LOCF)

Name of the Course : **B.Sc. (Programme) Mathematical Sciences / Physical Sciences**

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Marks are indicated.

1. (a) Determine where the graph of the function $f(x) = x^4 - 4x^3 + 10$ is concave up and where it is concave down. (6)

(b) Sketch the graph of the function

$y = \frac{1}{3}x^3 - 9x + 2.$ (6)

(c) (i) Evaluate $\lim_{x \to -\infty} \frac{(2x+5)(x-3)}{(7x-2)(4x+1)}$. (3)

(ii) Find all horizontal and vertical asymptotes of the graph of the function

$f(x) = 4 + \frac{2x}{x-3}$. (3)

2. (a) Find the critical points of the function $f(x) = x + \frac{1}{x}$ and identify each as a relative maximum, a relative minimum, or neither. (6.5)

(b) Evaluate the following limits using l'Hopital's rule

(i) $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ (3)

(ii) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$ (3.5)

(c) Trace the curve

$x = a(t + \sin t),\ y = a(1 - \cos t),\ -\pi \le t \le \pi.$ (6.5)

3. (a) Derive the reduction formula for $\int \sec^n x\, dx$ and hence evaluate $\int (a^2 + x^2)^{5/2} dx$. (6)

(b) Obtain the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$. Use it to evaluate $\int_0^{\pi/2} \sin^7 x \cos^4 x\, dx$. (6)

(c) Sketch the graph of $r = a(1 - \cos\theta)$ in polar coordinates, assuming 'a' to be a positive constant. (6)

4. (a) Find the volume of the solid generated when the region enclosed by $y = \sqrt{x+1}$, $y = \sqrt{2x}$ and $y = 0$ is revolved about the x axis. (6.5)

(b) Use cylindrical shells to find the volume of the solid generated when the region R bounded by the curve $y = x^2$ and the x-axis for $0 \le x \le 3$ is revolved-about the line $y = -2$. (6.5)

(c) Find the arc length of the parametric curve defined by

$x = \cos t + t \sin t,\ y = \sin t - t \cos t,\ 0 \le t < \pi.$ (6.5)

5. (a) Describe the graph of the equation :

$5x^2 + 9y^2 + 20x - 54y = -56.$ (6)

(b) Sketch the parabola and label the focus, vertex and directrix :

$$(x-1)^2 = 2\left(y-\frac{1}{2}\right).$$ (6)

(c) Rotate the coordinate axes to remove the xy-term. Identify the type of the conic and sketch its graph.

$$x^2 + 2\sqrt{3}\,xy + 3y^2 + 2\sqrt{3}\,x - 2y = 0.$$ (6)

6. (a) Sketch the graph of the hyperboloid of one sheet :

$$x^2 + y^2 - \frac{z^2}{4} = 1.$$ (6.5)

(b) Define the differentiability of a vector-valued function. If r(t) is a differentiable vector- valued function in 2-space or 3-space and $\|r(t)\|$ is constant for all t, then prove that r(t) and it's derivative are orthogonal vectors for all t. (6.5)

(c) Let $r_1(t) = (\tan^{-1} t)i + (\sin t)j + t^3k$ and

$$t_2(t) = (t^2 - t)i + (2t - 2)j + (\ln t)k$$

The graphs of $r_1(t)$ and $r_2(t)$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin. (6.5)

(200)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	:	**4668**	**E**
Unique Paper Code	:	32351202	
Name of the Paper	:	Differential Equations	
Name of the Course	:	**B.Sc. (Hons.) Mathematics**	
Semester	:	II	

Duration : 3 Hours — Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Use of non-programmable scientific calculator is allowed.

Section – 1

1. Attempt any **three** parts. Each part is of **5** marks.

   (a) Solve the initial value problem

   $$(2y\sin x\cos x + y^2\sin x)dx + (\sin^2 x - 2y\cos x)dy = 0, \quad y(0) = 3$$

   (b) Solve the differential equation

   $$(x^2 - 3y^2)dx + 2xy\,dy = 0$$

   (c) Solve the differential equation

   $$xy'' + 2y' = 6x$$

   (d) Solve the differential equation

   $$(x + 2y + 3)dx + (2x + 4y - 1)dy = 0$$

2. Attempt any **two** parts. Each part is of **5** marks.

   (a) In a certain cultnre of bacteria, the number of bacteria increased sixfold in 10 hours. How long did it take for the population to double?

(b) An arrow is shot straight upward from the ground with an initial velocity of 160 ft/s. It experiences both the deceleration of gravity and deceleration $v^2/800$ due to air resistance. How high in the air does it go?

(c) A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F. After 30 min the temperature of the cake is 140°F. When will it be 100°F?

Section – 2

3. Attempt any two parts. Each part is of **8** marks.

(a) The following differential equation describes the level of pollution in the lake

$$\frac{dC}{dt} = \frac{F}{V}(C_{in} - C),$$

where V is the volume, F is the flow (in and out), C is the concentration of pollution at time t and C_{in} is the concentration of pollution entering the lake. Let $V = 28 \times 10^6 m^3$, $F = 4 \times 10^6 m^3/month$. If only fresh water enters the lake,

i. How long would it take for the lake with pollution concentration $10^7 parts/m^3$ to drop below the safety threshold $4 \times 10^6 parts/m^3$?

ii. How long will it take to reduce the pollution level to 5% of its current level?

(b) In view of the potentially disastrous effects of overfishing causing a population to become extinct, some governments impose quotes which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h_0 X.$$

i. Show that the only non-zero equilibrium population is

$$X_c = K\left(1 - \frac{h}{r}\right).$$

ii. At what critical harvesting rate can extinction occur?

(c) In a simple battle model, suppose that soldiers from the red army are visible to the blue army, but soldiers from the blue army are hidden. Thus, all the red army can do is fire randomly into an area and hope they hit something. The blue army uses aimed fire.

i. Write down appropriate word equations describing the rate of change of the number of soldiers in each of the armies.

ii. By making appropriate assumptions, obtain two coupled differential equations describing this system.

iii. Write down a formula for the probability of a single bullet fired from a single red soldier wounding a blue soldier in terms of the total area A and the area exposed by a single blue soldier A_b.

iv. Hence write the rate of wounding of both armies terms of the probability and the firing rate.

Section – 3

4. Attempt any three parts. Each part is of **6** marks.

(a) Find the general solution of the differential

$$x^3y''' + 6x^2y'' + 4xy' = 0.$$

(b) Using the method of undetermined coefficients, solve the differential equation

$$y''' - 2y'' + y' = 1 + xe^x,\ y(0) = y'(0) = y''(0) = 1.$$

(c) Using the method of Variation of parameters, solve the differential equation

$$y'' + 3y' + 2y = 4e^x.$$

(d) Show that $y_1 = 1$ and $y_2 = \sqrt{x}$ are solutions of

$$yy'' + (y')^2 = 0,$$

but the sum $y = y_1 + y_2$ is not a solution. Explain why?

Section – 4

5. Attempt any **two** parts. Each part is of **8** marks.

(a) The pair of differential equations

$$\frac{dP}{dt} = rP - \gamma PT, \frac{dT}{dt} = qP,$$

where r, γ and q are positive constants, is a model for a population of microorganisms P, which produces toxins T which kill the microorganisms.

i. Given that initially there are no toxins and P_0 microorganisms, obtain an expression relating the population density and the amount of toxins.

ii. Hence, give a sketch of a typical phase-plane trajectory.

iii. Using phase-plane trajectory, describe what happens to the microorganisms over time.

(b) A model of a three species interaction is :

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ,$$

$$\frac{dY}{dt} = a_2XY - b_2Y,$$

$$\frac{dZ}{dt} = a_3XZ - b_3Z$$

Where a_i, b_i, c_i for $i = 1, 2, 3$ are all positive constants. Here $X(t)$ is the prey density and $Y(t)$ and $Z(t)$ are the two predator species densities.

i. Find all possible equilibrium populations. Is it possible for the three populations to coexist in equilibrium ?

ii. What does this suggest about introducing an additional predator into an ecosystem ?

(c) In a long range battle, neither army can see the other, but fires into a given area. A simple mathematical model describing this battle is given by the coupled differential equations

$$\frac{dR}{dt} = -c_1RB, \qquad \frac{dB}{dt} = -c_2RB$$

where c_1 and c_2 are positive constants.

i. Use the chain rule to find a relationship between R and B, given the initial numbers of soldiers for the two armies are r_0 and b_0, respectively.

ii. Draw a sketch of typical phase-plane trajectories.

iii. Explain how to estimate the parameter c_1 given that the blue army fires into a region of area A.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1271 **F**

Unique Paper Code : 2352571201

Name of the Paper : ELEMENTARY LINEAR ALGEBRA

Name of the Course : **B.Sc. (Prog.) DSC-B2**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting **two** parts from each question.

3. **All questions** carry equal marks.

P.T.O.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove th

$$\|x+y\| \le \|x\| + \|y\|$$

(b) Define norrn ofa vector. Find a unit vector in t

same direction as the vector $\left[\frac{1}{5}, -\frac{2}{5}, -\frac{1}{5}, \frac{1}{5},\right.$
Is the normalized (resulting) vector longer
shorter than the original? Why?

(c) Use Gaussi.an elimination method to solv
föllowing systems of linear equations. Give tl
complete solution set, and if the solution set
infinite, specify two panicular solutions.

$$\begin{aligned} 3x + 6y - 9z &= 15 \\ 2x + 4y - 6z &= 10 \\ -2x - 3y + 4z &= -6 \end{aligned}$$

2. (a) Determine whether the two matrices are row equivalent?

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 10 \\ 2 & 0 & 4 \end{bmatrix}$$

(b) Find. the rank of the following matrix.

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 4 & 0 & -2 & 1 \\ 3 & -1 & 0 & 4 \end{bmatrix}$$

(c) Express the vector X = [2, −5, 3] as a linear combination of the vectors a_1= [1, −3,2], a_2= [2, −4,−1], and a_3= [1, −5,7] if possible.

3. (a) Determine the characteristic polynomial of t
following matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(b) Show that the set of vectors of the form [a, b, c, a - 2b + c] in $\mathbb{R}^5$ forms a subspace of I
under the usual operations.

(c) For S = $\{x^3 + 2x^2, 1 - 4x^2, 12 - 5x^3, x^3 - x^2\}$, u
the Simplified Span Method to find a simplifie
general form for all the vectors in span(S), whe
S is the given subset of P_3 , the set of a
polynomials of degree less than or equal to 3 wi
real coefficients.

4. (a) Use the Independence Test Method to determin
whether the given set S is linearly independent o
linearly dependent.

s = {[1, -1, 0, 2], [0, -2, 1, 0], [2, 0, -1, 1]}

(b) Let the subspace W of $\mathbb{R}^5$ be the solution set to the matrix equation AX = 0 where A is

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 \\ 2 & -1 & 0 & 1 & 3 \\ 1 & -3 & -1 & 1 & 4 \\ 2 & 9 & 4 & -1 & -7 \end{bmatrix}$$

Find the basis and the dimension for W. Show that dim(W) + Rank(A) = 5.

(c) Show that P_n , the set of all polynomials of degree less than or equal to n with real coefficients, is a vector space under the usual operations of addition and scalar multiplication.

5. (a) Consider the mapping $f: R^3 \rightarrow R^3$ given by

$$f([a_1, a_2, a_3]) = [a_1, a_2, -a_3]$$

Prove that f is a linear transformation.

(b) Find the matrix for the linear transformation $L: P_3 \to R^3$ given by

$$L(a_3x^3 + a_2x^2 + a_1x + a_0) = [a_0 + a_1, 2a_2, a_3 - a_0]$$

With respect to the bases $B = (x^3, x^2, x, 1)$ for P_3 and $C = (e_1, e_2, e_3)$ for R^3

(c) Consider the linear operator $L: R^n \to R^n$ given by

$$L([a_1, a_2, \cdots, a_n]) = [a_1, a_2, 0 \cdots, 0]$$

Find the kernel of L and range of L.

6. (a) Consider the linear transformation $L: R^3 \to R^3$ given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the basis for kernel of L.

(b) Consider the linear operator $L: R^2 \to R^2$ given by

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Show that L is one-to-one and onto operator.

(c) Consider the linear transformation $L: P_3 \to P_2$ given by $L(p) = p'$ where $p \in P_3$

Is L an isomorphism?

[This question paper contains 8 printed pages.]

Your Roll No...............

F

Sr. No. of Question Paper : 2033

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : **GE**

Semester : II

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.

1. (a) If x and y are vectors in $\mathbb{R}^n$ then prove that

$$|x.y| \leq (||x||)(||y||).$$

P.T.O.

Also verify it for vectors x = [−1, −1, 0] and

$y = \left[\sqrt{2}, \sqrt{2}, \sqrt{2}\right]$.

(b) Use Gaussian Elimination to solve the following system of linear equations. Indicate whether the system is consistent or inconsistent. Give the complete solution set, if consistent.

$$3x - 3y - 2z = 23$$
$$-6x + 4y + 3z = -40$$
$$-2x + y + z = -12$$

(c) Use Gauss-Jordan row reduction method to find the complete solution set for the following system of equations.

$$4x - 8y - 2z = 0$$
$$3x - 5y - 2z = 0$$
$$2x - 8y + z = 0$$

2. (a) Define rank of a matrix. Find Rank of the matrix

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix}.$$

(b) Find all Eigen values corresponding to the matrix A. Also, find the eigenspace for each of the eigen value of the matrix A, where $A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$.

(c) Let $A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$. Determine whether the vector X = [5, 17, −20] is in row space of the matrix A. If so, then express X as a linear combination of the rows of A.

3. (a) Let V be a vector space. Let H and K be subspaces of V. Prove that $H \cap K$ is also a subspace of V. Give an example to show that $H \cup K$ need not be a subspace.

(b) Prove or disprove that the set S = { [1,2,1], [1,0,2], [1,1,0]} forms a basis of R^3.

(c) Use the Diagonalization Method to determine whether the matrix A is diagonalizable. If so, specify the matrices D and P and verify that $P^{-1}AP = D$, where

$$A = \begin{bmatrix} 19 & -48 \\ 8 & -21 \end{bmatrix}.$$

4. (a) (i) Let $S = \{(x,y,z): x + y = z \ \forall x, y, z \in R\}$. Show that S is a subspace of R^3.

(ii) Consider the vector space $M_{3\times3}(R)$ and the sets

T_1: the set of nonsingular 3×3 matrices

T_2: the set of singular 3×3 matrices.

Determine T_1 and T_2 are subspaces of $M_{3\times3}(R)$ or not?

(b) Define linearly independent subset of a finite dimensional vector space V. Use Independent Test Method to find whether the set S = {[1,0,1,2], [0,1,1,2], [1,1,1,3], [–1,2,3,1] } of vectors in R^4 is linearly independent or linearly dependent.

(c) Define basis of a vector space V. Show that the subset {[1,0,–1], [1,1,1],[1,2,4]} of R^3 forms a basis of R^3.

5. (a) (i) Determine whether the following function is a Linear Transformation or not?

L: $R^3 \to R^3$ given by

$L([x_1, x_2, x_3]) = [x_2, x_3, x_1]$.

(ii) Suppose L: $R^3 \to R^3$ is a linear operator and

L[1,0,0] = [–2,1,0], L[0,1,0] = [3,–2,1] and L[0,0,1] = [0,–1,3].

Find L[–3,2,4]. Give a formula for L[x, y, z], for any $[x, y, z] \in R^3$.

(b) Consider the linear Transformation L: $P_3(R) \to M_{2\times 2}(R)$ given by

$$L(ax^3 + bx^2 + cx + d) \begin{bmatrix} -3a-2c & -b+4d \\ 4b-c+3d & -6a-b+2d \end{bmatrix}.$$

Find the matrix for L with respect to the standard basis for P_3 and $M_{2\times 2}$. Also, Find the dimension of Ker(L) and Range(L).

(c) Let L: $P_3 \to P_4$ be given by

$$L(p) = \int p, \text{ for, } p \in P_3,$$

Where, $\int p$ represents the integration of p. Find the matrix for L with respect to the standard basis for P_3 and P_4. Use this matrix to calculate $L(4x^3 - 5x^2 + 6x - 7)$ by matrix multiplication.

6. (a) Let L: $R^3 \to R^3$ be given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(i) Is [1, –2, 3] in Ker(L)? Why or why not?

(ii) Is [2, –1, 4] in Range(L)? Why or why not?

(b) Consider the linear Transformation L: $M_{3\times 3} \to M_{3\times 3}$ given by

$$L(A) = A - A^T,$$

where, A^T represents the transpose of the matrix A. Find the Ker(L) and Range(L) of the Linear Transformation, and verify

$$\dim(\text{Ker}(L)) + \dim(\text{Range}(L)) = \dim(M_{3\times 3}).$$

(c) Suppose that L: $V \to W$ is a linear Transformation. Show that if $\{L(v_1), L(v_2), ..., L(v_n)\}$ is a linearly independent set of n distinct vectors in W, for some vectors $v_1, v_2, \ldots, v_n \in V$, then $\{v_1, v_2, \ldots, v_n\}$ is linearly independent set in V. Is the converse true? Justify with examples. Under what condition the converse holds true, Justify.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 6095 **E**

Unique Paper Code : 32355202

Name of the Paper : Linear Algebra

Name of the Course : **Generic Elective – Mathematics [other than Maths (H)]**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that

(i) $||x + y||^2 = ||x||^2 + ||y||^2$ if and only if $x . y = 0$.

P.T.O.

(ii) If $(x + y) . (x - y) = 0$, then $\|x\| = \|y\|$. (6½)

(b) If $x = [4, 0, -3]$ and $y = [3, 1, -7]$ be two vectors in $\mathbb{R}^3$, then decompose the vector y into two component vectors in directions parallel and orthogonal to the vector x. (6½)

(c) Define rank of a matrix. Also, find the rank of the matrix (6½)

$$\begin{bmatrix} 3 & 1 & 0 & 1 & -9 \\ 0 & -2 & 12 & -8 & -6 \\ 2 & -3 & 22 & -14 & -17 \end{bmatrix}$$

2. (a) Solve the following system of linear equations using Gaussian Elimination method

$$3x_1 - 6x_2 + 3x_4 = 9$$

$$-2x_1 + 4x_2 + 2x_3 - x_4 = -11$$

$$4x_1 - 8x_2 + 6x_3 + 7x_4 = -5$$

(6)

(b) Examine whether the matrix $A = \begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}$ is diagonalizable. (6)

(c) Let V be a vector space, and let W_1 and W_2 be subspaces of V. Show that

(i) $W_1 \cap W_2$ is also a subspace of V.

(ii) $W_1 + W_2$ defined by $W_1 + W_2 = \{w_1 + w_2 : w_1 \in W_1, w_2 \in W_2\}$ is also a subspace of V. (6)

3. (a) Use the Simplified Span Method to find a simplified general form for all the vectors in span(S) where

$$S = \left\{ \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$ is a subset of $\mathcal{M}_{22}$.

Is the set S linearly independent? Justify. (6½)

(b) Define a basis for a vector space. Examine whether the set

S = {[3, 1, −1], [5, 2, −2], [2, 2, −1]} forms a basis for $\mathbb{R}^3$? (6½)

(c) Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -3 \end{bmatrix}$. Using rank of A determine whether the homogeneous system AX = 0 has a non-trivial solution or not. If so, find the non-trivial solution. (6½)

4. (a) Let S = {[1, 0], [1, −3]} and T = {[1, −1], [1, 1]} be two ordered bases for $\mathbb{R}^2$. Find the transition matrix $P_{S \leftarrow T}$ from T-basis to S-basis. If v is in $\mathbb{R}^2$ and $[v]_T$ = [5,1], determine $[v]_S$. (6)

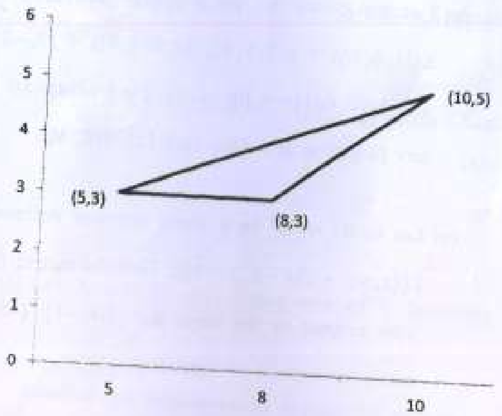
(b) Let L: $\mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator and L([1, 0, 0]) = [−2, 1, 0], L([0, 1, 0]) = [3, −2, 1] and L([0, 0, 1]) = [0, −1, 3]. Find L([x,y,z]) for any [x,y,z] ∈ $\mathbb{R}^3$. Also find L([−3, 2, 4]). (6)

(c) Let L: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear operator defined by L([x, y]) = [2x − y, x − 3y]. Find the matrix for L with respect to the basis T = {[4, −1], [−7, 2]} using the method of similarity. (6)

5. (a) For the graphic in the given figure, use ordinary coordinates in $\mathbb{R}^2$ to find new vertices after performing each indicated operation. Then sketch the figure that would result from this movement.

(i) Translation along the vector [2, −1]. (6½)

(ii) Reflection about the line y = 2x.



(b) State Dimension Theorem. Find a basis for Ker(L) and a basis for range(L) for the linear transformation L: $\mathbb{R}^3 \to \mathbb{R}^3$ given by (6½)

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \forall\, (x, y, z) \in \mathbb{R}^3$$

(c) Check whether the linear transformation L : $\mathcal{P}^3 \to \mathcal{P}^2$ defined by

$$L(a x^3 + b x^2 + c x + d) = a x^2 + b x + c$$

is an isomorphism or not. (6½)

6. (a) Find a least square solution for the following inconsistent system

$$\begin{pmatrix} 1 & -1 \\ 4 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}.$$ (6)

(b) Let W = span {[8, −1, −4], [4, 4, 7]} and v = [1,2,3] $\in \mathbb{R}^3$. Find $\text{proj}_W\, v$, and decompose v into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^\perp$. (6)

(c) Let W be the subspace of $\mathbb{R}^3$, whose vectors lie in the plane $2x - 5y + z = 0$. Find a basis for W and its orthogonal complement. (6)

May-June-2023

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1204 **F**

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : **B.Sc. (H) Mathematics**

Semester / Type : II / DSC

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $||x + y|| \le ||x|| + ||y||$. Also, verify the same for the vectors $x = [-1, 4, 2, 0, -3]$ and $y = [2, 1, -4, -1, 0]$ in $\mathbb{R}^5$.

(b) Using the Gauss – Jordan method, find the complete solution set for the following homogeneous system of linear equations:

$$4x_1 - 8x_2 - 2x_3 = 0$$

$$3x_1 - 5x_2 - 2x_3 = 0$$

$$2x_1 - 8x_2 + x_3 = 0$$

(c) Define the rank of a matrix. Using rank, find whether the non-homogeneous linear system AX = B, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

has a solution or not. If yes, find the solution.

2. (a) Consider the matrix :

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & -1 & 5 \\ -4 & -3 & 3 \end{pmatrix}$$

Determine whether the vector [4, 0, –3] is in the row space of A. If so, then express [4, 0, –3] as a linear combination of the rows of A.

(b) Consider the matrix :

$$A = \begin{pmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{pmatrix}$$

(i) Find the eigenvalue and the fundamental eigenvectors of A.

(ii) Is A diagonalizable? Justify your answer.

(c) Find the reduced row echelon form matrix B of the following matrix :

$$A = \begin{pmatrix} 1 & 2 & -2 & -11 \\ 2 & 4 & -1 & -10 \\ 3 & 6 & -4 & -25 \end{pmatrix}$$

and then give a sequence of row operations that convert B back to A.

3. (a) Let F_1 and F_2 be fields. Let $\mathcal{F}(F_1, F_2)$ denote the vector space of all functions from F_1 to F_2. A function $g \in \mathcal{F}(F_1, F_2)$ is called an even function if $g(-t) = g(t)$ for each $t \in F_1$ and is called an odd

function if $g(-t) = -g(t)$ for each $t \in F_1$. Prove that the set of all even functions in $\mathcal{F}(F_1, F_2)$ and the set of all odd functions in $\mathcal{F}(F_1, F_2)$ are subspaces of $\mathcal{F}(F_1, F_2)$.

(b) Let W_1 and W_2 be subspaces of a vector space V.

(i) Prove that $W_1 + W_2$ is a subspace of V that contains both W_1 and W_2.

(ii) Prove that any subspace of V that contains both W_1 and W_2 must also contain $W_1 + W_2$.

(c) (i) Let S_1 and S_2 are arbitrary subsets of a vector space V. Show that if $S_1 \subseteq S_2$ then span(S_1) $\subseteq$ span (S_2).

(ii) Let F be any field. Show that the vectors (1,1,0), (1,0,1) and (0,1,1) generate F^3.

4. (a) Define a linearly independent subset of a vector space V. Let $S = \{u_1, u_2, \ldots, u_n\}$ be a finite set of vectors. Prove that S is linearly dependent if and only if $u_1 = 0$ or $u_{k+1} \in$ span $(\{u_1, u_2, \ldots, u_k\})$ for some k, $(1 \le k < n)$.

(b) Let V be a vector space and $\beta = \{u_1, u_2, \ldots, u_n\}$ be a subset of V. Prove that β is a basis for V if and only if each $v \in V$ can be uniquely expressed as a linear combination of vectors of β, that is, can be expressed in the form $v = a_1u_1 + a_2u_2 + \ldots + a_nu_n$, for unique scalars $a_1, a_2, \ldots, a_n$.

(c) Let F be any field. Consider the following subspaces of F^5:

$W_1 = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 \mid a_1 - a_3 - a_4 = 0\}$

and

$W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 \mid a_2 = a_3 = a_4 = 0, a_1 + a_5 = 0\}$

Find bases and dimension for the subspaces W_1, W_2 and $W_1 \cap W_2$.

5. (a) Let V and W be vector spaces over a field F, and let $T : V \to W$ be a linear transformation. If $\beta = \{v_1, v_2, \ldots, v_n\}$ is a basis for V then prove that

$R(T) = \text{span } (T(\beta)) = \text{span}(\{T(v_1), T(v_2), \ldots, T(v_n)\})$

If T is one-to-one and onto then prove that $T(\beta) = \{T(v_1), T(v_2), \ldots, T(v_n)\}$ is a basis for W.

(b) Suppose that $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is linear,

$T(1,1) = (1,-2)$

$T(-1,1) = (2,3)$.

What is $T(-1,5)$ and $T(x_1, x_2)$?

Find $[T]_\beta^\gamma$ if $\beta = \{(1,1), (-1,1)\}$ and $\gamma = \{(1,-2), (2,3)\}$.

(c) For the following linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$:

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

find bases for null space N(T) and range space R(T). Also, verify the dimension theorem.

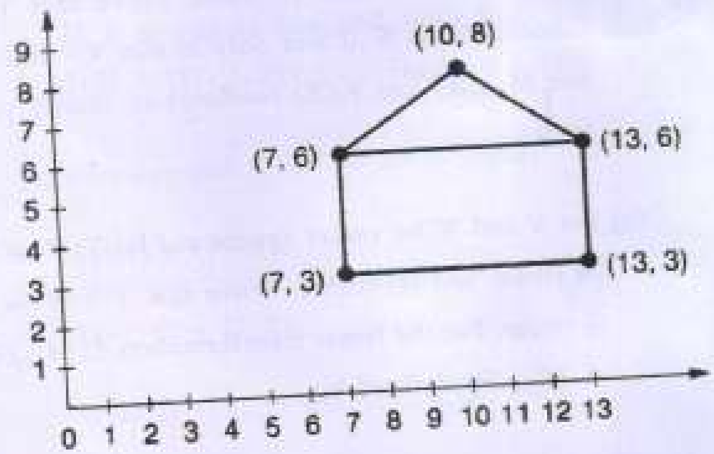
6. (a) Let V and W be finite dimensional vector spaces over the same field F. Then, prove that V is isomorphic to W if and only if dim V = dim W. Are $M_{2\times2}(\mathbb{R})$ and $P_3(\mathbb{R})$ isomorphic? Justify your answer.

(b) Let V and W be vector spaces and let $T: V \rightarrow W$ be linear and invertible. Prove that $T^{-1}: W \rightarrow V$ is linear. For the linear transformation $T: M_{2\times2}(\mathbb{R}) \rightarrow M_{2\times2}(\mathbb{R})$ defined by :

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & a \\ c & c+d \end{pmatrix}$$

determine whether T is invertible or not. Justify your answer.

(c) For the adjoining graphic, use homogenous co-ordinates to find the new vertices after performing scaling about (7,3) with scale factors of ½ in the x - direction and 3 in the y - direction.



Also, sketch the final figure that would result from this movement.

[This question paper contains 8 printed pages.]

Your Roll No...............

F

Sr. No. of Question Paper : 1242

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester / Type : II / DSC

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. Each part carries **7.5** marks.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Solve the initial value problem

$$(e^{2x}y^2 - 2x)\,dx + e^{2x}y\,dy = 0,\ y(0) = 2$$

(b) Solve

$(2x + \tan y)\, dx + (x - x^2 \tan y)\, dy = 0$

(c) Solve

(i) $(3x^2 + 4xy - 6)\, dy + (6xy + 2y^2 - 5)\, dx = 0$

(ii) $\frac{d^2y}{dx^2} = 2y\left(\frac{dy}{dx}\right)^3 = 2y$ by reducing the order.

2. (a) A certain rumor began to spread one day in a city with a population of 100,000. Within a week, 10,000 people had heard this rumor. Assume that the rate of increase of the number who have heard the rumor is proportional to the number who have not yet heard it. How long will it be until half the population has heard the rumor?

(b) The half-life of radioactive cobalt is 5.27 years. Suppose that a nuclear accident in a certain region has left the level of cobalt to be 100 times the acceptable level for habitation. How long will it be until the region is again habitable?

(c) A cake is removed from an oven at 210°F and left to cool at room temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F. What will be its temperature after 40 minutes? When will the temperature be 100°F?

3. (a) Show that the solutions x, x^2, x log x of the third order differential equation

$$x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$$

are linearly independent on $(0, \infty)$. Also find the particular solution satisfying the given initial condition.

$y(1) = 3,\ y'(1) = 2,\ y''(1) = 1$

(b) Solve the differential equation using the method of Variation of Parameters

$$\frac{d^2y}{dx^2} + 9y = \tan 3x$$

(c) Find the general solution of the differential equation using the method of undetermined Coefficients.

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 4e^{-x} + 3x^2$$

4. (a) Use the operator method to find the general solution of the following linear system

$$2\frac{dx}{dt} + \frac{dy}{dt} + x + 5y = 4t$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = 2$$

(b) Solve the initial value problem. Assume $x > 0$.

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3, \quad y(2) = 0, \quad y'(2) = 8$$

(c) A body with mass $m = \frac{1}{2}$ kg is attached to the end of a spring that is stretched 2m by a force of 16N. It is set in motion with initial position $x_0 = 1$m and initial velocity $v_0 = -5$m/s. Find the position function of the body as well as the amplitude, frequency and period of oscillation.

5. (a) Define the term Carrying Capacity. Derive the logistic equation

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right)$$

where K is the carrying capacity of the population. Also find the solution.

(b) The per-capita death rate for the fish is 0.5 fish per day per fish, and the per-capita birth rate is 1.0 fish per day per fish. Write a word equation describing the rate of change of the fish population. Hence obtain a differential equation for the number of fish. If the fish population at a given time is 240, 000, give an estimate of the number of fish born in one week.

(c) In an epidemic model where infected get recovered, the differential equation is of the form

$$\frac{dS}{dt} = -\beta SI, \ \frac{dI}{dt} = \beta SI - \gamma I$$

Use parameter values $\beta = 0.002$ and $\gamma = 0.4$, and assume that initially there is only one infective but there are 500 susceptibles. How many susceptibles never get infected, and what is the maximum number of infectives at any time? What happens as time progresses, if the initial number of susceptibles is doubled, $S(0) = 1000$? How many people were infected in total.

6. (a) A public bar opens at 6 p.m. and is rapidly filled with clients of whom the majority are smokers. The bar is equipped with ventilators that exchange the smoke-air mixture with fresh air. Cigarette smoke contains 4% carbon monoxide and a prolonged exposure to a concentration of more than 0.012% can be fatal. The bar has a floor area of 20m by 15m, and a height of 4m. It is estimated that smoke enters the room at a constant

rate of 0.006 m^3/min, and that the ventilators remove the mixture of smoke and air at 10 times the rate at which smoke is produced. The problem is to establish a good time to leave the bar, that is, sometime before the concentration of carbon monoxide reaches the lethal limit. Starting from a word equation or a compartmental diagram, formulate the differential equation for the changing concentration of carbon monoxide in the bar over time. By solving the equation above, establish at what time the lethal limit will be reached.

(b) Find the equilibrium solution of the differential equation

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right)$$

And discuss the stability of equilibrium solution.

(c) Consider a disease where the infected get recovered. A model describing this is given by the differential equations

$$\frac{dS}{dt} = -\beta SI, \ \frac{dI}{dt} = \beta SI - \gamma I$$

Use chain rule to find arelation between S and I. Obtain and sketch the phase-plane curves. Determine the direction of travel along the trajectories.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4548 E

Unique Paper Code : 32351201

Name of the Paper : Real Analysis (CBCS-LOCF)

Name of the Course : B.Sc. (Hons) Mathematics

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any two parts from each question.

1. (a) If x and y are positive real numbers with $x < y$, then prove that there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$. (6.5)

   (b) Define Infimum and Supremum of a nonempty set of $\mathbb{R}$. Find infimum and supremum of the set

   $$S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}.$$ (6.5)

P.T.O.

(c) State the completeness property of $\mathbb{R}$, hence show that every nonempty set of real numbers which is bounded below, has an infimum in $\mathbb{R}$. (6.5)

2. (a) Prove that there does not exist a rational number $r \in \mathbb{Q}$ such that $r^2 = 2$. (6)

(b) Define an open set and a closed set in $\mathbb{R}$. Show that if $a, b \in \mathbb{R}$, then the open interval (a, b) is an open set. (6)

(c) Let S be a nonempty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as: s \in S\}$. Prove that $\inf(aS) = a(\inf S)$ and $\sup(aS) = a(\sup S)$. (6)

3. (a) Define limit of a sequence. Using definition show that $\lim_{n\to\infty}\left(\frac{3n+1}{2n+5}\right) = \frac{3}{2}$. (6.5)

(b) Prove that every convergent sequence is bounded. Is the converse true? Justify. (6.5)

(c) Let $x_1 = 1$ and $x_{n+1} = \frac{1}{4}(2x_n + 3)$ for $n \in \mathbb{N}$. Show that $\langle x_n \rangle$ is bounded and monotone. Find the limit. (6.5)

4. (a) If $\langle a_n \rangle$ and $\langle b_n \rangle$ converges to a and b respectively, prove that $\langle a_n b_n \rangle$ converges to ab. (6)

(b) Show that $\lim_{n\to\infty} n^{1/n} = 1$. (6)

(c) State Cauchy Convergence Criterion for sequences. Hence show that the sequence $\langle a_n \rangle$, defined by $a_n = 1 + \frac{1}{2} + \cdots\cdots + \frac{1}{n}$, does not converge. (6)

5. (a) Prove that if an infinite series $\sum_{n=1}^{\infty} a_n$ is convergent then $\lim_{n\to\infty} a_n = 0$. Hence examine the convergence of $\sum_{n=1}^{\infty} \frac{n}{2n+3}$. (6.5)

(b) Examine the convergence or divergence of the following series.

(i) $\frac{2}{5} + \frac{4}{8} + \frac{6}{11} + \cdots$ (6.5)

(ii) $\sum_{n=1}^{\infty}\left(\frac{3n+5}{2n+1}\right)^{n/2}$

(c) Prove that $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}$, $p>0$ is convergent for $p>1$ and divergent for $p\le 1$. (6.5)

6. (a) State and prove ratio test (limit form). (6)

(b) Examine the convergence or divergence of the following series. (6)

(i) $\sum_{n=1}^{\infty}\frac{n^3+1}{n^4+3n^2+2n}$

(ii) $3+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}+\cdots$

(c) Prove that the series $\frac{1}{\ln 2}-\frac{1}{\ln 3}+\frac{1}{\ln 4}-\frac{1}{\ln 5}+\cdots$ is conditionally convergent. (6)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	:	**4530**	**E**
Unique Paper Code	:	32351401	
Name of the Paper	:	BMATH408- Partial Differential Equations	
Name of the Course	:	**B.Sc.(H) Mathematics**	
Semester	:	IV	

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All sections are compulsory.
3. Marks of each part are indicated.

Section – I

1. Attempt any two out of the following: [7.5+7.5]

(a) Find the integral surfaces of the equation $u\,u_x + u_y = 1$ for the initial data:

$$x(s,0) = s, y(s,0) = 2s, u(s,0) = s.$$

(b) Apply $\sqrt{u} = v$ and $v(x,y) = f(x) + g(y)$ to solve:

$$x^4 u_x^2 + y^2 u_y^2 = 4u.$$

(c) Find the solution of the initial–value systems

$$u_t + u\,u_x = e^{-x}v,\ v_t - av_x = 0,$$

with $u(x,0) = x$ and $v(x,0) = e^x$.

P.T.O.

Section – II

2. Attempt any one out of the following: [6]

(a) Derive the two- dimensional wave equation of the vibrating membrane

$$u_{tt} = c^2(u_{xx} + u_{yy}) + F,$$

where, $c^2 = T/\rho$, and T is the tensile force per unit length

$F = f/\rho$, and f be the external force, acting on the membrane.

(b) Drive the potential equation $\nabla^2 V = 0$, where ∇^2 is known as Laplace operator.

3. Attempt any two out of the following: [6+6]

(a) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

(b) Given that the parabolic equation

$$u_{xx} = a\, u_t + b\, u_x + c\, u + f,$$

where the coefficients are constants, by the substitution $u = v\, e^{\frac{1}{2}bx}$ and for the case $c = -(b^2/4)$, show that the given equation is reduced to the heat equation

$$v_{xx} = a\, v_t + g,$$

where $g = f\, e^{-bx/2}$.

(c) Reduce the equation

$$(n-1)^2 u_{xx} - y^{2n}\, u_{yy} = n\, y^{2n-1}\, u_y,$$

to canonical form for $n = 1$ and $n = 2$ if possible and also find their solutions.

Section – III

4. Attempt any three parts out of the following: [7+7+7]

(a) Determine the solution of the given below initial-value problem

$u_{tt} - c^2 u_{xx} = x, \quad u(x,0) = 0, \quad u_t(x,0) = 3.$

(b) Obtain the solution of the initial boundary-value problem

$u_{tt} = 9u_{xx}, \quad 0 < x < \infty, \; t > 0,$

$u(x,0) = 0, \quad 0 \le x < \infty,$

$u_t(x,0) = x^3, \quad 0 \le x < \infty,$

$u_x(0,t) = 0, \quad t \ge 0.$

(c) Solve:

$u_{tt} = c^2 u_{xx},$

$u(x,t) = f(x) \quad \text{on} \quad t = t(x),$

$u(x,t) = g(x) \quad \text{on} \quad x + ct = 0,$

where $f(0) = g(0)$.

(d) Determine the solution of the initial boundary-value problem:

$u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \; t > 0$

$u(x,0) = f(x), \quad 0 \le x \le l,$

$u_t(x,0) = g(x), \quad 0 \le x \le l,$

$u(0,t) = 0, \quad u(l,t) = 0, \quad t \ge 0.$

Section – IV

5. Attempt any three out of the following: [7+7+7]

(a) Determine the solution of the initial boundary value problem:

$u_t = 4\, u_{xx}, \quad 0 < x < 1, \; t > 0$

$u(x,0) = x^2\,(1-x), \quad 0 \le x \le 1,$

$u(0,t) = 0, \quad u(l,t) = 0, \quad t \ge 0.$

(b) Determine the solution of the initial boundary value problem by the method of separation of variables:

$u_{tt} = c^2 u_{xx}, \quad 0 < x < \pi, \; t > 0$

$u(x,0) = 0, \quad 0 \le x \le \pi,$

$u_t(x,0) = 8\sin^2 x, \quad 0 \le x \le \pi,$

$u(0,t) = 0, \quad u(\pi,t) = 0, \quad t \ge 0.$

P.T.O.

(c) Solve by using method of separation of variables:

$$u_{tt} - u_{xx} = h, \qquad 0 < x < 1,\ \ t > 0,\ h \text{ is a constant}$$
$$u(x,0) = x^2, \qquad 0 \le x \le 1,$$
$$u_t(x,0) = 0, \qquad 0 \le x \le 1,$$
$$u(0,t) = 0, \quad u(1,t) = 0, \qquad t \ge 0.$$

(d) State and prove the uniqueness of solution of the heat conduction problem.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 6207 E

Unique Paper Code : 32355402

Name of the Paper : GE-4: Numerical Methods

Name of the Course : CBCS / LOCF (Other than B.Sc. (H) Mathematics Hons.)

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Use of scientific calculator is allowed.

1. (a) Round-off the number 34.64867 correct up to three significant digits and then calculate absolute percentage error. (6)

P.T.O.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 6207 E

Unique Paper Code : 32355402

Name of the Paper : GE-4: Numerical Methods

Name of the Course : CBCS / LOCF (Other than B.Sc. (H) Mathematics Hons.)

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Use of scientific calculator is allowed.

1. (a) Round-off the number 34.64867 correct up to three significant digits and then calculate absolute percentage error. (6)

4\. (a) By using the initial solution (0,0,0), perform three iterations of the Gauss Seidel method for the following system of linear equations : (6.5)

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$

(b) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the given data and by using it estimate the value of f(3). (6.5)

x	1	2	4	8
f(x)	3	7	21	73

(c) Following table gives the amount of half yearly premium for policies maturing at different ages: (6.5)

Age (in years)	45	50	55	60	65
Premium (in Rs.)	114.84	96.16	83.32	74.48	68.48

Make the difference table. Obtain the forward Gregory-Newton interpolating polynomial and estimate the premium for policy maturing at the age of 46.

5. (a) For the following data, find f′(2) and f″(2) by using forward difference formulae

$$f'(x_i) \approx \frac{f(x_i + h) - f(x_i)}{h} \text{ and}$$

$$f''(x_i) \approx \frac{f(x_i) - 2f(x_i + h) + f(x_i + 2h)}{h^2}$$

x	0	1	2	3	4
f(x)	8	4	6	20	52

(6)

(b) For the function $f(x) = \ln x$, approximate $f'(2)$ by Richardson extrapolation using central difference formula $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$ with $h = 0.1$ and $h = 0.05$. (6)

(c) Use the formula

$$f'(x_i) \approx \frac{3f(x_i) - 4f(x_i - h) + f(x_i - 2h)}{2h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_i = \pi$, taking $h = 1, 0.1, 0.01$. (6)

6. (a) Approximate the value of $(\ln 2)^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Trapezoidal rule and Simpson's $\frac{1}{3}$ rule.

(6.5)

(b) Apply the Fleun method to approximate the solution of the initial value problem

$$\frac{dy}{dx} = \frac{1}{2}(1+x)y^2, \quad 0 \le x \le 1, \quad y(0) = 1,$$

by using 5 steps. (6.5)

(c) Given the initial value problem (IVP):

$$\frac{dy}{dx} = \frac{e^x}{y}, \ y(0) = 1,$$

Find y(0.5) and y(0.75) by using the modified Euler's method. Also find the absolute error at each step given, that the exact solution of the IVP is $y=\sqrt{2e^x-1}$. (6.5)

Sl No of QP	: 5713
Unique Paper Code	: 42354401
Name of the Paper	: Real Analysis
Name of the Course	: B.Sc. (Prog) Physical Sciences/Mathematical Sciences
Semester	: IV

Duration : 3 Hours Maximum Marks: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any **two** *parts from each question.*

1. (a) Define a countable set. Show that the set $\mathbb{Q}$ of rational numbers is countable.

(b) Define absolute value of a real number $'x'$.
Find all $x \in \mathbf{R}$ that satisfy the following inequalities:
(i) $4 < |x+2| + |x-1| < 5$
(ii) $|2x-1| \leq x+1$

(c) (i) Define supremum of a non-empty bounded subset S of $\mathbf{R}$.
(ii) Show that a real number u is the supremum of a non-empty subset S of $\mathbf{R}$ if and only if it satisfies the following conditions:
(1) $s \leq u$ for all $s \in S$.
(2) For each positive real number ε, there exists $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$.
(6,6)

2. (a) State the Archimedean Property of real numbers. Show that if $x \in \mathbf{R}$, then there exists a unique $n \in \mathbf{Z}$ such that $n - 1 \leq x < n$.

(b) Define the convergence of a sequence (x_n) of real numbers. Using the definition, evaluate the following limits:

(i) $\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n}$
(ii) $\lim_{n\to\infty} \left(\frac{(-1)^n n}{n^2+1}\right)$

(c) Let (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$, $\forall n \in N$. Show that the sequence $(\sqrt{x_n})$ converges to $\sqrt{x}$.

(6,6)

3. (a) Prove that every monotonically decreasing and bounded below sequence of real numbers converges.

(b) Show that the sequence (x_n) defined by
$x_1 = 1; x_{n+1} = \frac{1}{4}(2x_n + 3)$, $\forall n \geq 1$ is convergent. Also, find $\lim_{n\to\infty} x_n$.

(c) State Cauchy's Convergence Criterion for sequences of real numbers. Show directly from the definition that the following sequence is a Cauchy sequence:

$$\left(1+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}\right).$$

(6.5, 6.5)

4. (a) State and prove Comparison test for positive term series. Hence, show that the following series converges:

$$1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\cdots$$

(b) Suppose that (x_n) is a sequence of non-negative real numbers. Prove that the series $\sum x_n$ converges if and only if the sequence $S = (s_k)$ of partial sums is bounded.

(c) (1) State (without proof) D'Alembert's ratio test for an infinite series.

(2) Test for convergence the series:

(i) $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+---.$

(ii) $\frac{1}{\log 2}+\frac{1}{(\log 3)^2}+\frac{1}{(\log 4)^3}+\cdots$

(6.5, 6.5)

5. (a) (i) Define an absolutely convergent series. Is every convergent series absolutely convergent? Justify your answer.

(ii) Test for convergence the series:

(1) $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}---$

(2) $\sum_{n=1}^{\infty}(-1)^n.e^{-n}$

(b) Show that if $a > 0$, then the sequence $\left(\frac{nx}{1+n^2x^2}\right)$ converges uniformly on the interval $[a,\infty)$ but not uniformly on the interval $[0,\infty)$.

(c) State Weierstrass M-Test for uniform convergence of series. Hence, show that

$$\sum\frac{1}{x^2+n^2},\quad \forall x\in\mathbf{R}$$

is uniformly convergent.

(6.5,6.5)

6. (a) Find the radius of convergence and exact interval of convergence of the power series

$$\sum\frac{n+1}{(n+2)(n+3)}x^n.$$

(b) Show that the function $f(x)=x^2$ defined on the interval $[0,b]$, where $b>0$ is Riemann integrable.

(c) Show that every continuous function defined on $[a,b]$ is Riemann integrable.

(6.6)

[This question paper contains 2 printed pages.]

Your Roll No..............

Sr. No. of Question Paper	:	**4686**	**E**
Unique Paper Code	:	32351402	
Name of the Paper	:	Riemann Integration and Series of Functions	
Name of the Course	:	**B.Sc. (H) Mathematics**	
Semester	:	IV	

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **two** parts from each question.

1(a) Let f be integrable on [a, b], and suppose g is a function on [a, b] such that $g(x) = f(x)$ except for finitely many x in [a, b]. Show g is integrable and $\int_a^b g = \int_a^b f$ (6)

(b) Show that if f is integrable on [a, b] then f^2 also is integrable on [a, b]. (6)

(c) (i) Let f be a continuous function on [a,b] such that $f(x) \geq 0$ for all $x \in [a, b]$. Show that if $\int_a^b f(x)\,dx = 0$ then $f(x) = 0$ for all $x \in [a, b]$ (3)

(ii) Give an example of function such that $|f|$ is integrable on [0,1] but f is not integrable on [0,1]. Justify it. (3)

2(a) State and prove Fundamental Theorem of Calculus I. (6.5)

(b) State Intermediate Value Theorem for Integrals. Evaluate $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt$. (6.5)

(c) Let function $f: [0,1] \to R$ be defined as

$$f(x) = \begin{cases} x^2 & if\ x\ is\ rational \\ 0 & if\ x\ is\ irrational \end{cases}$$

Calculate the upper and lower Darboux integrals for f on the interval [0,1]. Is f integrable on [0,1] ? (6.5)

3(a) Examine the convergence of the improper integral $\int_0^\infty e^{-x}x^{n-1}dx$. (6)

(b) Show that the improper integral $\int_\pi^\infty \frac{Sinx}{x}\,dx$ is convergent but not absolutely convergent. (6)

P.T.O.

(c) Determine the convergence or divergence of the improper integral (6)

(i) $\int_0^1 \frac{dx}{x(\ln x)^2}$

(ii) $\int_1^\infty \frac{x\,dx}{\sqrt{x^3+x}}$

4(a) Show that the sequence

$$f_n(x) = \frac{nx}{1+nx},\ x \in [0,1],\ n \in N$$

converges non-uniformly to an integrable function f on [0,1] such that

$$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$$ (6.5)

(b) Show that the sequence $\{x^2 e^{-nx}\}$ converges uniformly on $[0, \infty)$. (6.5)

(c) Let (f_n) be a sequence of continuous function on $A \subset R$ and suppose that (f_n) converges uniformly on A to a function $f: A \longrightarrow R$. Show that f is continuous on A. (6.5)

5(a) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \geq 0$. Show that sequence (f_n) converges non-uniformly on $[0, \infty)$ and converges uniformly on $[a, \infty), a > 0$. (6.5)

(b) State and prove Weierstrass M-test for the uniform Convergence of a series of functions. (6.5)

(c) Show that the series of functions $\sum \frac{1}{n^2+x^2}$, converges uniformly *on R* to a continuous function. (6.5)

6(a) (i) Find the exact interval of convergence of the power series (3)

$$\sum_{n=0}^{\infty} 2^{-n} x^{3n}$$

(ii) Define $sinx$ as a power series and find its radius of convergence (3)

(b) Prove that $\sum_{n=1}^{\infty} n^2 x^n = \frac{x(x+1)}{(1-x)^3}$ for $|x| < 1$ and hence evaluate $\frac{\sum_{n=1}^{\infty} n^2(-1)^n}{3^n}$. (6)

(c) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ have radius of convergence $R > 0$. Then f is differentiable on $(-R, R)$ and

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \quad for\ |x| < R.$$ (6)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4810 **E**

Unique Paper Code : 32351403

Name of the Paper : Ring Theory & Linear Algebra – I

Name of the Course : **B.Sc. [Hons.] Mathematics CBCS (LOCF)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

P.T.O.

1. (a) Find all the zero divisors and units in $\mathbb{Z}_3 \oplus \mathbb{Z}_6$. (6)

(b) Prove that characteristic of an integral domain is 0 or prime number p. (6)

(c) State and prove the Subring test (6)

2. (a) Let R be a commutative ring with unity and let A be an ideal of R then prove that R/A is a field if and only if A is a maximal ideal of R. (6)

(b) Let A and B are two ideals of a commutative ring R with unity and A+B=R then show that $A \cap B = AB$. (6)

(c) If an ideal I of a ring R contains a unit then show that I=R. Hence prove that the only ideals of a field F are {0} and F itself. (6)

3. (a) Find all ring homomorphism from $\mathbb{Z}_6$ to $\mathbb{Z}_{15}$. (6.5)

(b) Let $R = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$ and Φ be the mapping that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ to a-b. Show that

(i) Φ is a ring homomorphism.

(ii) Determine Ker Φ.

(iii) Show that R/Ker Φ is isomorphic to $\mathbb{Z}$. (6.5)

(c) Using homomorphism, prove that an integer n with decimal representation $a_k\, a_{k-1} \dots \dots a_0$ is divisible by 9 iff $a_k + a_{k-1} + \dots + a_0$ is divisible by 9. (6.5)

4. (a) Let V(F) be the vector space of all real valued function over $\mathbb{R}$.

Let $V_e = \{f \in V \mid f(x) = f(-x)\ \forall\ x \in \mathbb{R}\}$

and $V_o = \{f \in V \mid f(-x) = -f(x)\ \forall\ x \in \mathbb{R}\}$

Prove that V_e and V_o are subspaces of V and $V = V_e \oplus V_o$. (6)

(b) Let V(F) be a vector space and let $S_1 \subseteq S_2 \subseteq V$. Prove that

(i) If S_1 is linearly dependent then S_2 is linearly dependent

(ii) If S_2 is linearly independent then S_1 is linearly independent (6)

(c) Show that $S = \left\{\begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}\right\}$ forms a basis for $M_{2X2}(\mathbb{R})$. (6)

5. (a) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3).$$

Find Null space and Range space of T and verify Dimension Theorem. (6.5)

(b) Define T: $M_{2X2}(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b) + (2d)x + bx^2$

Let $\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and $\gamma = \{1, x, x^2\}$ be basis of $M_{2X2}(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively. Compute $[T]_{\beta}^{\gamma}$. (6.5)

(c) Let V and W be vector spaces over F, and suppose that $\{v_1, v_2, \ldots, v_n\}$ be a basis for V. For $w_1, w_2, \ldots, w_n$ in W. Prove that there exists exactly one linear transformation T: V → W such that $T(v_i) = w_i$ for $i = 1, 2, \ldots, n$. (6.5)

6. (a) Let T be the linear operator on $\mathbb{R}^2$ define by

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}$$

Let β be the standard ordered basis for $\mathbb{R}^2$ and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}.$$

Find $[T]_{\beta'}$. (6.5)

(b) Let V and W be finite dimensional vector spaces with ordered basis β and γ respectively. Let T: V → W be linear. Then T is invertible if and only if $[T]_{\beta}^{\gamma}$ is invertible.

Furthermore, $\left[T^{-1}\right]_{\gamma}^{\beta} = \left(\left[T\right]_{\beta}^{\gamma}\right)^{-1}$. (6.5)

(c) Let V, W and Z be finite dimensional vector spaces with ordered basis α, β, γ respectively. Let T: V → W and U: W → Z be linear transformations.

Then $[UT]_{\alpha}^{\gamma} = [U]_{\beta}^{\gamma}[T]_{\alpha}^{\beta}$. (6.5)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	:	**4628**	**E**
Unique Paper Code	:	32353401	
Name of the Paper	:	SEC 2-Computer Algebra Systems and Related Softwares	
Name of the Course	:	**CBCS-LOCF-B.Sc. (H) Mathematics**	
Semester	:	IV	

Duration : 2 Hours

Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has four questions in all.
3. All questions are compulsory.
4. Use anyone of the **CAS := Mathematica/Maple/Maxima/any other** to answer the questions

Q1. Attempt both parts (i) and (ii).

(i) Fill in the blanks $1 \times 5 = 5$

a. The line numbers assigns to the input as ....
b. You can obtain information about a specific command by typing.........
c. The command to calculate π to 100 decimal places is ....
d. Lines are terminated by .... to suppress the output
e. The command to calculate the binomial coefficient $\binom{7}{2}$ is ....

(ii) Explain any **FIVE** of the following 'R' commands in short : $1 \times 5 = 5$

a. qqnorm( )
b. read.csv( )

c. ls( )
d. rm( )
e. as.character( )
f. tail( )

Q 2. Write a short note on any four from the following: $2\times 4 = 8$

(i) How to include exclusions and gridlines in a plot in any CAS. Also explain the difference between them.

(ii) How to put a logarithmic scale on horizontal axis in a plot of 2^x in any CAS.

(iii) How to sketch a contour plot in any of the CAS.

(iv) How to plot 3-dimensional surface in any CAS. Explain it by an example.

(v) How to form a new matrix from two existing matrices of same order by stacking them on top of each other in any CAS.

(vi) Explain the rules for defining a function in any CAS.

Q3. Do any four from the following: $2\times 4 = 8$

(i) Write the command for sketching the curve
$y = (x-1)^2, \quad -2 \le x \le 2,$ with colour of the curve blue

(ii) Write the command for plotting the graph of the following:
$y = e^x \cos x, \quad z = e^x \sin x, \quad 0 \le x \le 5.$

(iii) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 5 & 1 \end{bmatrix}$

Write commands for generating:

a. Eigen values and Eigen vectors of Matrix A
b. Diagnolise the matrix

(iv) Write the output of the following commands in the statistical software 'R'

```
>Garden=c(47,19,50,46,9,4)
>Hedgerow=c(10,3,0,16,3,0)
>Parkland=c(40,5,10,8,0,6)
>Pasture= c(2,0,7,4,0,0)
>data=c(Garden, Hedgerow, Parkland, Pasture, Woodland)
>bird=matrix(data,ncol=5,dimnames=list(c('Blackbird','Chaffinch','Great
Tit','House Sparrow','Robin','Song Thrush'), c('Garden', 'Hedgerow', 'Parkland',
'Pasture')))
>bird
```

(v) Write command for solving the system of equations:

$$x + 2y + 3z = 2;\ \ x - y + 3z = 0;\ \ 2x + 3y - 4z = 2$$

Q4. Attempt any three parts from the following: (4X3=12)

(i) Write code of the following in software- R:

a. Make a list "L1" in software R, containing following vectors: V1 = {p, q, r, s}, V2 = {2, 3, 4, 5}and V3 = {1.5, 3.5, 8.5}.

b. Write code to extract V2 from L1

c. Find square root of the mean of V2.

d. Add V4 = {3, 7, 5, 11} at third position in the list L1.

(ii) Consider the following dataframe object 'x':

	C1	C2	C3	C4	C5
R1	8	15	53	28	1
R2	23	7	35	55	9
R3	7	9	1	6	17
R4	11	14	56	3	32
R5	9	2	12	45	5
R6	12	18	9	3	18

Write code of the following in software- R:

a. Find the column means and column sums of 'x'.

b. Find the minimum and maximum values of the dataframe 'x'.

c. Create a scatter chart of 'x'.

(iii) Explain difference between the following in software-R

a. **as.data.frame()** and **data.frame()** commands.

b. matrix and data fame.

c. **order()** and **rank()** commands.

d. vector and list.

(iv) Write possible R commands for the following questions:

	C1	C2	C3	C4	C5
R1	12	5	35	8	12
R2	13	27	32	5	5
R3	21	10	11	16	19
R4	5	11	16	10	23
R5	13	2	12	42	5
R6	10	14	8	20	30

a. Create the above matrix 'z'.
b. Extract second column of 'z'.
c. Make Histogram of row "R2".
d. Convert this matrix into dataframe.
e. Find standard deviation of vector "R3" of the converted dataframe

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4512 **E**

Unique Paper Code : 32351601

Name of the Paper : BMATH 613 – Complex Analysis

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt **two** parts from each question.

1. (a) Find and sketch, showing corresponding orientations, the images of the hyperbolas

$x^2 - y^2 = c_1$ $(c_1 < 0)$ and $2xy = c_2$ $(c_2 < 0)$

under the transformation $w = z^2$. (6)

P.T.O.

(b) (i) Prove that the limit of the function

$$f(z)=\left(\frac{z}{\bar{z}}\right)^2$$

as z tends to 0 does not exist.

(ii) Show that

$$\lim_{z\to\infty}\frac{4z^2}{(z-1)^2}=4.$$ (3+3=6)

(c) Show that the following functions are nowhere differentiable.

(i) $f(z) = z - \bar{z}$,

(ii) $f(z) = e^y \cos x + ie^y \sin x$. (3+3=6)

(d) (i) If a function f(z) is continuous and nonzero at a point z_0, then show that $f(z) \neq 0$ throughout some neighborhood of that point.

(ii) Show that the function $f(z) = (z^2 - 2)e^{-x}e^{-iy}$ is entire. (3+3=6)

2. (a) (i) Write $|\exp(2z + i)|$ and $|\exp(iz^2)|$ in terms of x and y. Then show that

$|\exp\ \exp\ (2z+i) + \exp(iz^2)| \le e^{2x} + e^{-2xy}$.

(ii) Find the value of z such that

$$e^z = 1+\sqrt{3}i$$ (3.5+3=6.5)

(b) Show that

(i) $\overline{\cos(iz)} = \cos(i\bar{z})$ for all z;

(ii) $\overline{\sin(iz)} = \sin(i\bar{z})$ if and only if $z = n\pi i$ $(n = 0 \pm 1, \pm 2, \ldots)$. (3.5+3=6.5)

(c) Show that

(i) $\log\log(i^2) = 2\log i$ where

$$\log z = \ln r + i\theta\left(r > 0, \frac{\pi}{4} < \theta < \frac{9\pi}{4}\right).$$

(ii) $\log\log(i^2) \neq 2\log i$ where

$$\log z = \ln r + i\theta\left(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4}\right).$$

(3.5+3=6.5)

(d) Find all zeros of sinz and cosz. (3.5+3=6.5)

3. (a) State Fundamental theorem of Calculus.

Evaluate the following integrals to test if Fundamental theorem of Calculus holds true or not :

(i) $\int_0^{\pi/2} \exp(t+it)dt$

(ii) $\int_0^1 (3t-i)^2 dt$ (2+2+2=6)

(b) Let y(x) be a real valued function defined piecewise on the interval $0 \le x \le 1$ as

$y(x) = x^3\sin(\pi/x),\ 0 < x \le 1$ and $y(0) = 0$

Does this equation $z = x + iy,\ 0 \le x \le 1$ represent

(i) an arc

(ii) A smooth arc. Justify.

Find the points of intersection of this arc with real axis. (2+2+2=6)

(c) For an arbitrary smooth curve C: $z = z(t),\ a \le t \le b$, from a fixed point z_1 to another fixed point z_2, show that the value of the integral depends only on the end points of C.

State if it is independent of the arc under consideration or not?

Also, find its value around any closed contour. (3+1+2=6)

(d) Without evaluation of the integral, prove that

$\left|\int_C \frac{1}{z^2+1} dz\right| \le \frac{1}{2\sqrt{5}}$ where C is the straight line segment from 2 to 2 +i. Also, state the theorem used. (4+2=6)

4. (a) Use the method of antiderivative to show that

$\int_C (z-z_0)^{n-1} dz = 0,\ n = \pm1, \pm2, \ldots$ where C is any closed contour which does not pass through the point z_0. State the corresponding result used. (4+2.5=6.5)

(b) Use Cauchy Gourset theorem to evaluate :

(i) $\int_C f(z)dz$, when $f(z) = \frac{1}{z^2+2z+2}$ and C is the unit circle $|z| = 1$ in either direction.

(ii) $\int_C f(z)dz$, when $f(z)=\frac{5z+7}{z^2+2z-3}$ and C is the circle $|z-2| = 2$. (3+3.5=6.5)

(c) State and prove Cauchy Integral Formula. (2+4.5=6.5)

(d) Evaluate the following integrals :

(i) $\int_C \frac{\cos z}{z(z^2+8)}dz$, where C is the positive oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$.

(ii) $\int_C \frac{2s^2-s-2}{s-2}ds$, $|z| \neq 3$ at $z = 2$, where C is the circle $|z| = 3$. (3.5+3=6.5)

5. (a) If a series of complex numbers converges then prove that the nth term converges to zero as n tends to infinity. Is the converse true? Justify. (6.5)

(b) Find the Maclaurin series for the function $f(z) = \sinh z$. (6.5)

(c) If a series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges to f(z) at all points interior to some circle $|z - z_0| = R$, then prove that it is the Taylor series for the function f(z) in powers of $z - z_0$. (6.5)

(d) Find the integral of f(z) around the positively oriented circle $|z| = 3$ when $f(z)=\frac{(3z+2)^2}{z(z-1)(2z+5)}$. (6.5)

6. (a) For the given function $f(z)=\left(\frac{z}{2z+1}\right)^3$, show any singular point is a pole. Determine the order of each pole and find the corresponding residue. (6)

(b) Find the Laurent Series that represents the function $f(z)=z^2 \sin\frac{1}{z^2}$ in the domain $0 < |z| < \infty$. (6)

(c) Suppose that $z_n = x_n + iy_n$, $(n = 1,2,3,...)$ and $S = X + iY$. Then show that

$$\sum_{n=1}^{\infty} z_n = S \text{ iff } \sum_{n=1}^{\infty} x_n = X \text{ and } \sum_{n=1}^{\infty} y_n = Y.$$

(6)

(d) If a function f(z) is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then show that

$$\int_c f(z)dz = 2\pi i\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right].$$

(6)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4749 **E**

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (H) Mathematics CBCS (LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.
4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

1. (a) Explain Duration of a zero-coupon bond. A 5-year bond with a yield of 12% (continuously compounded) pays a 10% coupon at the end of each year.

(i) What is the bond's price?

(ii) Use duration to calculate the effect on the bond's price of a 0.1% decrease in its yield? (You can use the exponential values: e^x = 0.8869, 0.7866, 0.6977, 0.6188, and 0.5488 for x = –0.12, –0.24, –0.36, –0.48, and –0.60, respectively)

(b) Portfolio A consists of 1-year zero coupon with a face value of ₹2000 and a 10-year zero coupon bond with face value of ₹6000. Portfolio B consists of a 5.95-year zero coupon bond with face value of ₹5000. The current yield on all bonds is 10% per annum.

(i) Show that both portfolios have the same duration.

(ii) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(You can use the exponential values: e^x = 0.905, 0.368, 0.552, 0.861, 0.223 and 0.409 for x = –0.1, –1.0, –0.595, –0.15, –1.5 and –0.893 respectively)

(c) Explain difference between Continuous Compounding and Monthly Compounding. What rate of interest with continuous compounding is equivalent to 15% per annum with monthly compounding?

(d) (i) "When the zero curve is upward sloping, the zero rate for a particular maturity is greater than the par yield for that maturity. When the zero curve is downward sloping the reverse is true." Explain.

(ii) Why does loan in the repo market involve very little credit risk?

2. (a) Explain Hedging. How is the risk managed when Hedging is done using?

(i) Forward Contracts; (ii) Options

(b) (i) Suppose that a March call option to buy a share for ₹50 costs ₹2.50 and is held until March. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised?

(ii) It is May, and a trader writes a September put option with a strike price of ₹20. The stock price is ₹18, and the option price is ₹2. Describe the trader's cash flows if the option is held until September and the stock price is ₹25 at that time.

(c) Write a short note on European put options. Explain the payoffs in different types of put option positions with the help of diagrams.

(d) (i) A trader writes an October put option with a strike price of ₹35. The price of the option is ₹6. Under what circumstances does the trader make a gain.

(ii) A company knows that it is due to receive a certain amount of a foreign currency in 6 months. What type of option contract is appropriate for hedging?

3. (a) Draw the diagrams illustrating the effect of changes in volatility and risk-free interest rate on both European call and put option prices when $S_0 = 50$, $K = 50$, $r = 5\%$, $\sigma = 30\%$, and $T = 1$.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the vega of a European call and the vega of a European put on a non-dividend-paying stock.

(c) A European call option and put option on a stock both have a strike price of ₹20 and an expiration date in 3 months. Both sell for ₹3. The risk-free interest rate is 10% per annum, the current stock

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price is ₹19, and a ₹1 dividend is expected in 1 month. Identify the arbitrage opportunity open to a trader? ($e^{-0.0083} = 0.9917$)

(d) Find lower bound and upper bound for the price of a 1-month European put option on a non-dividend-paying stock when the stock price is ₹30, the strike price is ₹34, and the risk-free interest rate is 6% per annum? Justify your answer with no arbitrage arguments. ($e^{-0.005} = 0.9950$)

4. (a) Consider the standard one-period model where the stock price goes from S_0 to S_0u or S_0d with $d < 1 < u$, and consider an option which pays f_u or f_d in each case, and assume that the interest rate is r and time to maturity is T. Derive the formula for the no-arbitrage price of the option.

(b) A stock price is currently ₹40. It is known that at the end of one month it will be either ₹42 or ₹38. The risk-free interest rate is 6% per annum with continuous compounding. Consider a portfolio consisting of one short call and Δ shares of the stock. What is the value of Δ which makes the portfolio riskless? Using no-arbitrage arguments, find the price of a one-month European call option with a strike price of ₹39? (You can use exponential value: $e^{0.005} = 1.005$)

(c) Construct a two-period binomial tree for stock an European call option with

$S_0 = ₹100, u = 1.3, d = 0.8, r = 0.05, T = 1$ year, $K = ₹9$

and each period being of length $\Delta t = 0.5$ year. Find the price of the European call. If the call was American, will it be optimal to exercise the option early? Justify your answer. ($e^{-0.025} = 0.9753$)

(d) What do you mean by the volatility of a stock? How can we estimate volatility from historical prices of the stock?

5. (a) Let S_0 denote the current stock price, σ the volatility of the stock, r be the risk-free interest rate and T denote a future time. In the Black-Scholes model, the stock price S_T at time T in the risk-neutral world satisfies

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right]$$

where $\phi(m, v)$ denotes a norma distribution with mean m and variance v.

Using risk-neutral valuation, derive the Black-Scholes formula for the price of a European call option on the underlying stock S, strike price K and maturity T.

(b) A stock price follows log normal distribution with an expected return of 16% and a volatility of 35%. The current price is ₹38. What is the probability that a European call option on the stock with an exercise price of ₹40 and a maturity date in six months will be exercised? (You can use values: $\ln(38) = 3.638$, $\ln(40) = 3.689$)

(c) What is the price of a European call option on a non-dividend-paying stock when the stock price is ₹69, the strike price is ₹70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?

(You can use exponential values: $e^{-0.0144} = 0.9857$, $e^{-0.025} = 0.9753$)

(d) A stock price is currently ₹40. Assume that the expected return from the stock is 15% and that its volatility is 25%. What is the probability distribution for the rate of return (with continuous compounding) earned over a 2-year period?

6. (a) Discuss theta of a portfolio of options and calculate the theta of a European call option on a non-dividend-paying stock where the stock price is ₹49, the strike price is ₹50, the risk-free interest rate is 5% per annum and the time to maturity is 20 weeks, and the stock price volatility is 30% per annum. ($\ln(49/50) = -0.0202$)

(b) (i) Explain stop-loss hedging scheme.

(ii) What does it mean to assert that the delta of a call option is 0.7? How can a short position in 1,000 options be made delta neutral when the delta of each option is 0.7?

(c) Find the payoff from a butterfly spread created using call options. Also draw the profit diagram corresponding to this trading strategy.

(d) Companies X and Y have been offered the following rates per annum on a ₹5 million 10-year investment :

	Fixed rate	Floating rate
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

Company X requires a fixed-rate investment; Company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and that will appear equally attractive to X and Y.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4874 **E**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF)– B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Solve the following Linear Programming Problem by Graphical Method :

Maximize $2x + y$

subject to $x + 2y \leq 10$

$x + y \leq 6$

$x - y \leq 2$

$x - 2y \leq 1$

$x \geq 0, y \geq 0.$

(b) Define a Convex Set. Show that the set S defined as :

$S = \{(x, y) \mid y^2 \geq 4ax;\ x \geq 0,\ y \geq 0\}$ is a Convex Set.

(c) Find all basic feasible solutions of the equations :

$x_1 + 2x_2 + 4x_3 + x_4 = 7$

$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

(d) Prove that to every extreme point of the feasible region, there corresponds a basic feasible solution of the Linear Programming Problem :

Minimize $z = cx$

subject to $Ax = b,\ x \geq 0$

2\. (a) Let us consider the following Linear Programming Problem :

Minimize $z = cx$

subject to $Ax = b\ ,\ x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution corresponding to a basis B where $x_B = B^{-1}b$. Suppose $z_0 = c_B x_B$ is the value of objective function such that $z_j - c_j \leq 0$ for every column a_j in A. Show that z_0 is the minimum value of z of the problem and that the given basic feasible solution is optimal feasible solution.

(b) Let $x_1 = 1, x_2 = 1, x_3 = 1$ be a feasible solution to the system of equations :

$x_1 + x_2 + 2x_3 = 4$

$2x_1 - x_2 + x_3 = 2$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Using Simplex Method, find the solution of the following Linear Programming Problem :

Maximize $5x_1 + 4x_2$

subject to $x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 4$

$5x_1 + x_2 \leq 15$

$x_1, x_2 \geq 0.$

(d) Solve the following Linear Programming Problem by Big - M Method :

Maximize $-x_1 - x_2 + x_3$

subject to $x_1 - x_2 - x_3 = 1$

$-x_1 + x_2 + 2x_3 \geq 1$

$x_1, x_2, x_3 \geq 0.$

3. (a) Solve the following Linear Programming Problem by Two Phase Method :

Maximize $-x_1 + 3x_2$

subject to $x_1 + x_2 \geq 1$

$-2x_1 + 3x_2 \leq 6$

$x_1 \geq 0,\ x_2 \geq 0.$

(b) Find the solution of given system of equations using Simplex Method :

$5x_1 + 2x_2 = 14$

$2x_1 + x_2 = 6$

Also find the inverse of A where $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

(c) Find the solution of the following Linear Programming Problem :

Minimize $3x_1 + 2x_2$

subject to $-x_1 + x_2 \leq 1$

$5x_1 + 3x_2 \leq 15$

$x_1 \geq 0,\ x_2 \geq 3/2.$

(d) Find the optimal solution of the Assignment Problem with the following cost matrix :

Job \ Machines	I	II	III	IV	V	VI
A	3	4	6	5	4	9
B	5	4	9	7	6	10
C	8	7	6	5	4	6
D	7	4	5	11	10	4
E	5	6	7	8	5	9
F	4	3	5	6	7	4

4. (a) Find the dual of the following Linear Programming Problem :

Maximize $3x_1 + 4x_2 - 3x_3$

Subject to $x_1 - 2x_2 + 5x_3 \geq 2$

$3x_1 + 7x_2 - 4x_3 = -8$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ is unrestricted.

(b) Prove that if the Primal Problem has a finite optimal solution then the Dual also has a finite optimal solution and the two optimal objective function values are equal.

(c) Using Complementary Slackness Theorem, find optimal solutions of the following Linear Programming Problem and its Dual.

Minimize $2x_1 + 15x_2 + 5x_3 + 6x_4$

subject to $x_1 + 6x_2 + 3x_3 + x_4 \geq 2$

$-2x_1 + 5x_2 - x_3 + 3x_4 \leq -3$

$x_1, x_2, x_3, x_4 \geq 0.$

(d) For the following cost minimization Transportation Problem find initial basic feasible solutions by using North West Comer rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions (in terms of the cost) :

Destination / Source	A	B	C	D	E	Supply
I	10	10	11	12	10	20
II	13	14	11	15	10	35
III	11	10	17	12	15	40
Demand	25	10	30	15	15	

5. (a) Solve the following cost minimization Transportation Problem :

Destinations / Origin	I	II	III	IV	Availability
A	14	11	13	12	22
B	13	17	10	15	15
C	13	15	16	14	8
Requirements	7	12	17	9	

(b) A University wish to allocate four subjects and six teachers claim that they have the required knowledge to teach all the subjects. Each subject can be assigned to one and only one teacher. The cost of Assignment of subject to each teacher is given in table below. Allocate the subjects to appropriate faculty members for optimal Assignment. Also find two Teachers who are not assigned any course.

Teachers \ Subjects	A	B	C	D
I	28	47	36	38
II	36	43	43	46
III	43	38	36	33
IV	47	48	31	38
V	48	38	41	43
VI	43	52	43	49

(c) Define rectangular fair Game. Using Maxmin and Minmax Principle, find the maximum pay-off for player 1 will have and minimum pay-off for player 2 for the following pay-off matrix :

Player 1

$$\text{Player 2}\begin{bmatrix} 10 & 8 & 4 \\ 9 & -5 & 15 \\ -1 & 7 & 6 \end{bmatrix}.$$

(d) Convert the following Game Problem into a Linear Programming Problem for player A and player B and solve it by Simplex Method :

Player B

$$\text{Player A}\begin{bmatrix} 6 & 3 & 5 \\ 2 & 5 & 1 \end{bmatrix}.$$

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This paper contains 3 printed pages

Unique Paper Code : 42357602

Name of the Paper : DSE- Probability and Statistics

Name of the Course : CBCS-LOCF: B.SC. Physical Sciences/Mathematical Sciences

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your roll number on the top immediately on receipt of this question paper.
2. Attempt all the six questions.
3. Each question has three parts. Attempt any two parts from each question.
4. Each part in Question 1, 3, 5 carries 6 marks.
5. Each part in Question 2, 4, 6 carries 6.5 marks.
6. Use of scientific calculator is allowed.

1. a) i) If A and B are events in the sample space S, then prove that :
$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$
ii) Let X be a random variable with Distribution Function F_X . Prove that for $a < b, P[a < X \leq b] = F_X(b) - F_X(a)$.

b) Show that $f(x) = \begin{cases} e^{-x} & ; 0 < x < \infty \\ 0 & ; elsewhere \end{cases}$ represents a probability density function. Also calculate P(X > 1)

c) i) Show that if X is a random variable and a, b are constants, then
$$E[(aX+b)^n] = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(X^{n-i}).$$
ii) Let the probability mass function $P(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere. If $P(0) = \frac{1}{4}$, find $E[X^2]$.

2. a) The random variable X has the probability distribution :
$f(x) = \frac{1}{8}\binom{3}{x}$ $for\ x = 0, 1, 2\ and\ 3.$
Find the moment generating function of this random variable and use it to determine $\mu_1'\ and\ \mu_2'$, where $\mu_r' = E[X^r]$ is the rth moment about the origin.

b) Show that the mean and the variance of the binomial distribution are $\mu = n\theta$ and $\sigma^2 = n\theta(1-\theta)$.

c) Find the probabilities that a random variable having the standard normal distribution will take on a value:
i) less than 1.30.
ii) less than −0.25.
iii) between 0.45 and 1.30.

3. a) Show that the normal distribution has:

i) a relative maximum at x = μ.

ii) inflection points at x = $\mu - \sigma$ and x = $\mu + \sigma$.

b) Let $f(x_1, x_2) = \begin{cases} 4x_1x_2 & : 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & ; \quad otherwise \end{cases}$ be the pdf of X_1 and X_2.

Find $P(0 < X_1 < \frac{1}{2}, \frac{1}{4} < X_2 < 1)$ and $P(X_1 < X_2)$.

c) Let the random variable X and Y have joint probability mass function as:

(x, y)	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$
$P(x,y)$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

i) Find marginal probability mass function of X and Y.

ii) $P(X + Y \leq 2)$.

4. a) Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = \begin{cases} 2 & ; 0 < x_1 < x_2 < 1 \\ 0 & ; otherwise \end{cases}$

i) Find conditional pdf $f_{1|2}(x_1|x_2)$.

ii) Find conditional mean $E(X_1|x_2)$ and the conditional variance $Var(X_1|x_2)$.

b) Let $f(x/y) = \begin{cases} \frac{cx}{y^2} & ; \quad 0 < x < y, 0 < y < 1 \\ 0 & ; \quad elsewhere \end{cases}$

be the conditional density of (X, Y) and

$f_2(y) = \begin{cases} ky & : 0 < y < 1 \\ 0 & : elsewhere \end{cases}$ be the marginal pdf of Y. Determine:

i) the constants c and k.

ii) joint pdf of X and Y.

iii) $P\left[\frac{1}{4} < X < \frac{1}{2} \middle| Y = \frac{5}{8}\right]$

iv) $P\left[\frac{1}{4} < X < \frac{1}{2}\right]$

c) Consider a random experiment that consists of drawing at random one chip from a bowl containing 10 chips of the same shape and size. Each chip has an ordered pair of numbers on it: one with (1, 1), one with (2, 1), two with (3, 1), one with (1, 2), two with (2, 2), and three with (3, 2). Let the random variables X_1 and X_2 be defined as the respective first and second values of the ordered pair. Find the joint probability mass function p (x_1, x_2) of X_1 and X_2, provided with p (x_1, x_2) equal to zero elsewhere.

5. a) If two random variables X and Y have the joint density given by

$$f(x,y) = \begin{cases} e^{-x-y} & ; \quad x > 0, y > 0 \\ 0 & ; \quad elsewhere \end{cases}$$

Show that $M(t_1, t_2) = (1-t_1)^{-1}(1-t_2)^{-1}; t_1, t_2 < 1$.

Also show that $[e^{tx+ty}] = (1-t)^{-2}; t < 1$.

b) Using method of least squares to fit a straight line for the following data:

X	1	2	3	4	5
Y	5	7	9	10	11

c) If X and Y have Joint pdf-

$$f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0 ; & otherwise \end{cases}$$

Find the correlation coefficient between X and Y.

6. a) If X and Y have Joint pdf

$$f(x,y) = \begin{cases} 3x & ; 0 < y < x, \ 0 < x < 1 \\ 0 & ; \quad otherwise \end{cases}$$

Are X and Y are independent? If not, find $E[Y|X]$.

b) If $X_i, i = 1, 2, 3, 4, \dots, 10$ be independent random variables, each being uniformly distributed over $(0, 1)$. Estimate $P(\sum_1^{10} X_i > 7)$.

c) A die is thrown 3600 times, show that the number of sixes lies between 550 and 650 is at least $\frac{4}{5}$.

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[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4792 **E**

Unique Paper Code : 32351602

Name of the Paper : Ring Theory and Linear Algebra – II

Name of the Course : **B.Sc. (H) Mathematics (CBCS – LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated

1. (a) (i) Prove that If F is a field, then F[x] is a Principal Ideal Domain.

   (ii) Is $\mathbb{Z}[x]$, a Principal Ideal Domain? Justify your answer.

   (b) Prove that $<x^2 + 1>$ is not a maximal ideal in $\mathbb{Z}[x]$.

P.T.O.

(c) State and prove the reducibility test for polynomials of degree 2 or 3. Does it fail in higher order polynomials? Justify. (4+2,6,6)

2. (a) (i) State and prove Gauss's Lemma.

(ii) Is every irreducible polynomial over $\mathbb{Z}$ primitive? Justify.

(b) Construct a field of order 25.

(c) In $\mathbb{Z}\left[\sqrt{(-5)}\right]$, prove that $1+3\sqrt{(-5)}$ is irreducible but not prime. (4+2.5,6.5,6.5)

3. (a) Let $V = \mathbb{R}^3$ and define $f_1, f_2, f_3 \in V^*$ as follows :

$f_1(x, y, z) = x - 2y,$

$f_2(x, y, z) = x + y + z,$

$f_3(x, y, z) = y - 3z.$

Prove that $\{f_1, f_2, f_3\}$ is a basis for V^* and then find a basis for V for which it is the dual basis.

(b) Test the linear operator $T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$, $T(f(x)) = f(0) + f(1)(x + x^2)$ for diagonalizability and if diagonalizable, find a basis β for V such that $[T]_\beta$ is a diagonal matrix.

(c) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$. Find an expression for A^n where n is an arbitrary natural number. (6,6,6)

4. (a) For a linear operator $T: \mathbb{R}^3 \to \mathbb{R}^3$, $T(a, b, c) = (-b + c, a + c, 3c)$, determine the T-cyclic subspace W of $\mathbb{R}^3$ generated by $e_1 = (1, 0, 0)$. Also find the characteristic polynomial of the operator T_W.

(b) State Cayley-Hamilton theorem and verify it for the linear operator $T:P_2(\mathbb{R}) \to P_2(\mathbb{R})$, $T(f(x)) = f'(x)$.

(c) Show that the vector space $\mathbb{R}^4 = W_1 \oplus W_2 \oplus W_3$ where $W_1 = \{(a, b, 0, 0): a, b \in \mathbb{R}\}$, $W_2 = \{(0,0, c, 0): c \in \mathbb{R}\}$ and $W_3 = \{(0,0,0, d): d \in \mathbb{R}\}$. (6.5,6.5,6.5)

5. (a) Consider the vector space $\mathbb{C}$ over $\mathbb{R}$ with an inner product $<.,.>$. Let $\bar{z}$ denote the conjugate of z. Show that $<.,.>'$ defined by $<z, w>' = <\bar{z}, \bar{w}>$ for all $z, w \in \mathbb{C}$ is also an inner product on $\mathbb{C}$. Is $<.,.>''$ defined by $<z, w>'' = <z + \bar{z}, w + \bar{w}>$ for all $z, w \in \mathbb{C}$ an inner product on $\mathbb{C}$? Justify your answer.

(b) Let $V = P(\mathbb{R})$ with the inner product $<p(x), q(x)> = \int_{-1}^{1} p(t)q(t)dt \ \forall p(x), q(x) \in V$. Compute the orthogonal projection of the vector $p(x) = x^{2k-1}$ on $P_2(\mathbb{R})$, where $k \in \mathbb{N}$.

(c) (i) For the inner product space $V = P_1(R)$ with $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)dt$ and the linear operator T on V defined by $T(f) = f' + 3f$, compute $T^*(4 - 2t)$.

(ii) For the standard inner product space $V = \mathbb{R}^3$ and a linear transformation $g: V \to \mathbb{R}$ given by $g(a_1, a_2, a_3) = a_1 - 2a_2 + 4a_3$, find a vector $y \in V$ such that $g(x) = \langle x, y\rangle$ for all $x \in V$. (6,6,2+4)

6. (a) Prove that a normal operator T on a finite-dimensional complex inner product space V yields an orthonormal basis for V consisting of eigenvectors of T. Justify the validity of the conclusion of this result if V is a finite-dimensional real inner product space.

(b) Let $V = M_{2\times2}(\mathbb{R})$ and $T: V \to V$ be a linear operator given by $T(A) = A^T$. Determine whether T is normal, self-adjoint, or neither. If possible, produce an orthonormal basis of eigenvectors of T for V and list the corresponding eigenvalues.

(c) For the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ find an orthogonal matrix P and a diagonal matrix D such that $P^*AP = D$. (6.5,6.5,6.5)

(1000)